M. CHERKASHENKO

ON THE THEORY OF SYNTHESIS OF MINIMAL SCHEMES OF SYSTEMS CONTROL OF HYDRAULIC AND PNEUMATIC DRIVES

Showed the strict compliance of the scientific direction "Synthesis of minimum control schemes of hydraulic and pneumatic drive systems" developed by the author with the point of view of general algebra, algebra of logic, graph theory and automata theory. The synthesis of the minimum graph of operations, which is a mathematical model of the control system, has been proved. The legitimacy of the methods of undivided decomposition of equations describing the scheme of the control system has been proved. The control system is considered as a cyclic Moore finite automaton. By a cyclic automaton (CA) we will understand the mathematical model of a device designed to control cyclic processes, which are a set of technological operations performed in a certain sequence. In this regard, the automaton at each clock necessarily passes into some new state, and for a finite number of cycles the target reaches any state, and its graph contains a contour, covering all states. In general, the CA may contain several circuits, so that each circuit is interpreted either as one of the possible sequences of technological operations due to the corresponding mode of operation, or as an independent and simultaneous execution of a number of sets of technological operations. A sequential decomposition of the CA is presented in order to represent it by the sequential operation of automata with one internal state. Such a consideration of the function of transitions will naturally lead to a decrease in the number of elements in the implementation of the CA. The study will be subjected to the CA, the graph of which consists of a single circuit, since the results obtained are easily generalized to multi-circuit CA. Obtaining a breakdown of the states of a cyclic automaton in the manner indicated above is performed directly according to any automaton description without any additional calculations, tables and other constructions.

Keywords: mathematical model, general algebra, graph theory, automata theory, minimal scheme, equations.

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DO ТЕОРІЇ СИНТЕЗУ МІНІМАЛЬНИХ СХЕМ СИСТЕМ УПРАВЛІННЯ ГІДРО-І ПНЕМОПРОВІДІВ

Показано сувору відповідність розробленого автором наукового напряму «Синтез мінімальних схем управління систем гідро- та пневмопровідів» з погляду загальної алгебри, алгебри логіки, теорії графів та теорії автоматів. Доведено синтез мінімального графа операцій, що є математичною моделлю системи керування. Доведено правомірність методів нероздільної декомпозиції рівнянь, що описують схему системи управління. Система управління сприймається як циклічний кінцевий автомат Мура. Під циклічним автоматом (ЦА) розуміється математична модель пристрою, принаймнішими для управління циклічними процесами, які є суккупністю технологічних операцій, які виконуються в певній послідовності. В зв'язку з цим, автомат у кожному такті неодмінно перетворюється на деякий новий стан, причому за кінцеве число тактів ЦА досягає будь-якого стану, а граф його містить контур, що охоплює всі стани. У загальному випадку ЦА може містити кілька контурув, оскільки кожен контур інтерпретується як одна з можливих послідовностей виконання технологічних операцій, обумовлена відповідним режимом роботи, або як незалежне та одночасне виконання низки суккупностей технологічних операцій.

Представлена послідовна декомпозиція ЦА з метою представлення його послідовної роботи в елементах з одним внутрішнім станом. Такий перегляд функції переходів природно призводить до зменшення кількості елементів і модулів, отже кожен контур інтерпретується як незалежна операція, або як одна з можливих послідовностей виконання технологічних операцій.

Ключові слова: математична модель, загальна алгебра, теорія графів, теорія автоматів, мінімальна схема, рівняння.

Introduction. In the synthesis of systems of hydraulic and pneumatic automatics, a standard positional structure is used, which has known advantages, the main disadvantage of which is the complexity of the schemes. Partial minimization of the standard positional structure was proposed in the works of Yuditsky S.A., Goedcke W., Belforte G., Reizo J., etc. [1]. The method of complete minimization of the standard positional structure was first published by the author in [1, 2]. It is based on the synthesis of the minimum graph of operations and the synthesis of equations with using the proposed mathematical model of the "correspondence matrix" [2]. In the synthesis of schemes, methods of separate decomposition of equations are used, they are described in the works of Yuditsky S.A., Bettini A., Middleton F., Gauthier D., Eng B., Rohner P. et al. [1]. The main disadvantage of these methods is the complexity of the schemes. For the first time, the principles of undivided decomposition of equations and the method of synthesizing circuits on switchgear were presented by the author in [1, 3]. Methods of undivided decomposition of equations lead to minimum schemes, they are based on the decomposition of the equation into two variables, the selection of decomposition variables and the calculation of residual functions, which leads, in combination with a modular element base [3], to schemes with a minimum number of modules and elements.

In this article, the author focuses on the strict combination of the developed scientific direction "Synthesis of minimum control schemes of hydro- and pneumatic drive systems" with the point of view and view of general algebra, graph theory and automata theory.

Under the cyclic automaton (CA) we will understand the mathematical model of a device designed to control cyclic processes, which are a set of technological operations performed in a certain sequence. In this regard, the automaton on each clock cycle necessarily goes into some new state, and for a finite number of cycles the CA reaches any state, and its graph contains a contour [4, 5], covering all states. In general, CA can contain several contours, so that each circuit is interpreted either as one of the possible sequences of technological operations due to the corresponding mode of operation, or as an independent and simultaneous performance of a number of sets of technological operations. A sequential decomposition of the CA is presented in order to represent it by the sequential operation of automata with one internal state. Such a consideration of the function of transitions will naturally lead to a decrease in the number of elements in the implementation of the CA. The study will be subjected to the CA, the graph of which consists of a single circuit, since the results obtained are easily generalized to multi-circuit CA. Obtaining a breakdown of the states of a cyclic automaton in the manner indicated above is performed directly according to any automaton description without any additional calculations, tables and other constructions.

Keywords: mathematical model, general algebra, graph theory, automata theory, minimal scheme, equations.

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the memory elements (EP); \( X = \{x_1, x_2, ..., x_p\} \) – input alphabet, whose signals come from sensors that monitor the state of the IS; \( Z = \{z_1, z_2, ..., z_m\} \) – the output alphabet whose signals affect the inputs of the IS; for any state \( y_i \in Y \) and input word \( pi \in X \) consisting of a set of input signals, here \( i \in \{1, 2, ..., n\}, \delta = \delta(y_i, pi) \) – the function of transitions; \( \lambda = \lambda(y_i, pi) \) – shifted output function.

Here is a sequential decomposition of the CA in order to represent it by the sequential operation of automata with one internal state so that \( a_0, a_2, ..., a_k \)

\[ a_n = (X_0, b_0); a \in \{1, 2, ..., t\}; \]

\[ X_0 = Y_0 \times X; Y = (\prod_i Y_i) \times X \rightarrow Z. \]

Then \( A = (\prod_i Y_i, X, Z, b, \lambda) \), where \( \delta(a_i, p_i) = \delta_n(a, p) \); \( P_0 \) – a set of signals that cause the transition. Such a consideration of the function of transitions will naturally lead to a decrease in the number of elements in the implementation of CA. The study will be subjected to the CA, the graph of which consists of a single circuit, since the results obtained are easily generalized to multichannel CA. Consider the set of S transitions into the set of states \( Y \) of the automaton \( A \). By selecting any state as the initial state, \( y_i \in Y \); you can record transitions as follows:

Starting position

\[ y_1 \rightarrow y_2 \]

\[ y_3 \rightarrow y_4 \]

\[ \quad \ldots \]

\[ y_{n+1} \rightarrow y_1 \]

Then arbitrary transition \( s; y_1 \rightarrow y_2; y_2 \rightarrow y_3; \ldots ; y_{n+1} \rightarrow y_1 \) is the forming element, and, denoting simply \( sj \) – s, write the sequence as \( s, ss, s_2, \ldots , s_n \) and when the superposition operation is maintained, the homomorphism condition is satisfied

\[ f_1(s^1) = z^1z^2 = f_1(s^1). \]

Since \( z^1 \) can be equal \( z^2 \) (due to possible equality \( Q_1 \) and \( Q_2 \)), the isomorphism condition is not satisfied

\[ f_1(s^1) \neq f_1(s^2) \mbox{ at } s^1 \neq s^2. \]

Each transition \( s \) of the automaton \( A \) corresponds to the input word \( P_1 \). It is not difficult to show by analogy with the set \( B \) that the set \( D: q, x, x^2, ..., x^{n-1} \) – a superposition monoid with a forming element \( x \) that shows the change in input words during the transition \( s \), the unit element \( q \) is the initial input word. Monoid \( B \) is uniquely mapped to monoid \( D \) so that

\[ f_2: e, s, s^2, s^3, ..., s^{n-1}, q, x, x^2, ..., x^{n-1} \]

and when the superposition operation is maintained, the homomorphism condition is satisfied. Since \( x^1 \) can be equal \( x^2 \) (due to the possible equality of the input words \( P_1 \) and \( P_2 \)) the isomorphism condition is not satisfied. Thus, monoid \( B \) is homomorphically displayed on monoids \( C \) and \( D \).

Set of transitions of an automaton \( A \) as a monoid by a superposition operation allows you to consider the work of the automaton as a sequential operation of the automata \( a_1, a_2, ..., a_C \).

Consider a \( C_\alpha \) subset of monoid \( C \) of successive changes in output words. The subset \( Z_\alpha \subset Z \) it induces should not contain signals to turn on and off the same actuator.

In the monoid \( B \), the subset \( B_C \) and \( C_\alpha \) correspond to the subset \( D_\alpha \), since \( Q_1 \neq Q_2 \), \( Q_2 \subset Z_\alpha \), these subsets are isomorphic. In the monoid \( D \), the subsets \( B_C \) and \( C_\alpha \) correspond to the subset \( D_\alpha \). The latter does not induce the appearance of identical input words, since the output words of the subset repeat the positions of the actuators (the position of the actuators is controlled by the final switches, etc., from which the input signals. Hence, subsets \( B_\alpha \), \( C_\alpha \) and \( D_\alpha \) are isomorphic. A subset of states \( Y_{\alpha} \subset Y \), to which transitions of the subset \( B_\alpha \) lead determines the division \( \pi \) of the states of the automaton \( A \) into equivalence classes by the consistent output words of the subset, such that \( Z_\alpha \pi = \{Y_{\alpha}\} \), \( \forall Y_{\alpha} \), \( Y_{\alpha} \cap Y_{\beta} \neq 0 \) at \( \alpha \neq \beta \).

Obtaining the partitioning of the states of a cycle automaton by the above method is not laborious, and can
be performed directly from any automaton description without any additional calculations, tables and other constructions.

**Synthesis minimal graph operations.** Method full minimize described. In the first step of designing determine the number of internal states, which equals the number of conduction elements of the storage unit. To do this, we carry out the partition \( \pi \) sequence of input vectors \( P \) \( CA \) into disjoint subsets — blocks \( B \) such that \( U_B P = P \), and \( B_0 \cap B_P = 0 \). In the case of multi scheme graph operations for each scheme partition is carried out separately. The same set \( P_z \), that cause different output sets \( CA_z \) and \( z_P \) by partitioning must be in different blocks and not be the last elements of the neighboring blocks \( B_0 \) and \( B_{i+1} \) (following the first unit is considered the last). In addition, any set \( P_z \) the next block should not be identical to the last set of the previous block \( B_0 \).

The last statement can be represented as two adjacent blocks where \( \{\ldots, P_z + 1 \rightarrow z, \ldots, P \rightarrow z, \ldots\} \). Assuming this arrangement sets in order to reduce the number of blocks \( |B| \), it is necessary to introduce an additional delay in release synthesis \( z_t \).

Let us illustrate the partition sets \( P \) by constructing a graph partitioning \( G_z \). Incorporating memory element meets the last item \( P_z \) block \( B_0, P \rightarrow S_0 \) (\( S \) — multiple functions memory element inclusions). Each vertex of \( G_z \) unit is responsible \( B_0 \) and encompassed the loop when the corresponding block has more than one element. If the graph \( G_z \) it comprises two peaks, one control automaton is applied using two inverse outputs. Arcs of the graph which are not loops forming one loop \( G_{in} \) if a partition may comprise one unit. In the opposite case, the graph \( G_{z} \) group contains loops.

Minimal graph operations and the proof of its minimal.

**Theorem.** The partition graph \( G_z \) is realized by a standard positional structure if identical combinations \( P_z \) producing different combinations \( y_0 \) and \( z_\mu \) are assigned to different nonadjacent arcs.

In fact, to distinguish \( CA \) transitions in which two identical sets act \( P_z \), that cause different sets \( z_\mu \) and \( z_\mu \) it can only be an extension of their signal outputs signals with memory element (ME) \( y_0 \) and \( y_\mu \) \( (\alpha \neq \beta; y_\alpha, y_\beta \in Y; Y \rightarrow \beta\) ME plurality of outputs). Partitions \( \pi \) it is constructed in such a way that on the graph \( G_z \) one internal state \( \alpha \) (block \( B_0 \)) corresponds to an arc that is not a loop, and its preceding adjacent loop. Hence, for the two transitions \( A/\alpha_{\gamma_{i+1}} \) and \( A/A_{i+1} \) column \( G_z \), which belong to the loop or noise and following its adjacent arc of the graph \( G_z \), under the action sets in a sequence identical \( S_{\gamma_{i+1}}(P_z \rightarrow z_\gamma) \), which corresponds to the transition \( A/\alpha_{\gamma_{i+1}} \) and \( S_{\gamma_{i+1}}(P_z \rightarrow z_\gamma) \), what corresponds to the transition \( A/A_{i+1} \), \( P_{\gamma_{i+1}} = P_{\gamma_{i+1}} \), as \( P_{i+1} = P_i \). Those initial set of \( CA \) \( z_\mu \) will appear in the transition \( A/A_{i+1} \). That occurs in the cycle before the transition \( A/A_{i+1} \).

If the same set \( P_\gamma \) and \( P_\gamma \) they are located on contiguous arcs \( \alpha \) and \( \alpha \mu + 1 \) column \( G_z \), that are not loops, is considering a similar sequence \( S_{\gamma_{i+1}}(P_\gamma \rightarrow z_\gamma) \), we obtain extension \( P_{\gamma_{i+1}} \) and \( P_{\gamma_{i+1}} \), but as \( P_{\gamma_{i+1}} \) responsible shutdown memory element \( \alpha + 1 \), \( P_{\gamma_{i+1}} = \gamma_{i+1} \), memory element \( \alpha + 2 \), and \( P_{i+1} = P_i \) (recall that in the case of standard positional structure, disconnection of any preceding memory element off next output), we obtain “slip” status \( \alpha + 2 \) by state \( \alpha + 1 \), which is contrary to the work cycle \( CA \) (does not meet the stability condition). Absence of identical input sets \( CA \) on contiguous arcs of operations that are not loops corresponds to satisfying a second condition for correctness generalized vertices of operations, namely the implementation of stability.

If the same set \( P_\gamma \) located on the arc \( \alpha \), that is not a loop and its adjacent loop trail \( \alpha + 1 \) column \( G_z \), the elongation \( P_{\gamma_{i+1}} \) which belongs to the arc \( \alpha \) in sequence \( S_{\gamma_{i+1}} \), responsible memory element \( \alpha + 1 \). The sequence \( S_{\gamma_{i+1}} \) elongation \( P_{\gamma_{i+1}} \). It belongs to the loop \( \alpha + 1 \). But as \( P_{\gamma_{i+1}} \) met the inclusion memory element \( \alpha + 1 \), the transition \( A/A_{i+1} \) column \( G \) set \( P_{i+1} \) sequentially operates with signals \( y_\mu \) and \( y_{\mu+1} \), i.e. in transition \( A/A_{i+1} \) appears premature \( CA \) \( z_\mu \) which corresponds to the transition \( A/A_{i+1} \). It is obvious that such an arrangement is only possible if \( z_\mu = \sum_{\nu} (x_\nu = z_\nu) \), i.e. if in the transitions, respectively, \( A/A_{i+1} \) and \( A/A_{i+1} \), on off one actuator (actuator switched on and off). The assertion is proved.

Thus, the arcs of the graph \( G_z \) that are not loops, recorded last elements of blocks, on loops – an ordered set of the remaining elements.

**Undivided decomposition methods.** Consider undivided decomposition methods developed by M. Cherkashenko (i.e. undivided implement of the function specified in the disjunctive normal form), including automated scheme syntheses that makes maximum use of logical and functional capabilities of the selected basic apparatus (modules and components) and lead to the minimum structure.

In developing hydropneumatic automation system designers face the challenge of creating a minimum of the number of logic elements. Thus solved a lot of problems, namely an increase in reliability, cost reduction, reduction in size, increase in speed, simplification of installation and commissioning, simplify the operation of the system as a whole. Here, the author suggests some developed algorithms that allow the implementation hydropneumatic automation scheme using the most frequently used in the practice of their creation – distribution equipment.

The equation for the output of the distributor can be represented as follows:

\[
z = x_1 + x_2 + (x_1 + x_2) y_4.
\]

It should be noted that when \( x_1 = 0 \), \( z = x_1 y_4 + x_2 y_3 \).

It will be shown that at the outlet may be implemented of 55 logical functions. The repetition-free basis (with equal ease of use of the direct and inverse values of the arguments) such apparatus provides the following functions and works sum of three arguments.

It is necessary to make an important statement. Practice shows construction hydropneumatic automation schemes implementing logic functions nonrepeating expediently carried out separately decomposition.
methods, functions and implementation repetition of arguments in different terms is advantageously carried out by methods undivided decomposition.

Realization of schemes by using undivided decomposition pneumatic distributors associated with decomposition of logic functions of two variables. For more than a simple expansion designed multifunctional logic modules, which in turn are universal for repetition-free functions. Consider the generalized uses of logic modules to implement pneumatic distributors schemes on distribution apparatus.

The function at the output of the module is of the form

\[ z = \overline{x} f_0(0, 0) + \overline{x} x f_1(0, 1) + x f_2(1, 0) + \overline{x} x f_3(1, 1). \] (2)

Such a function is formula decomposition of logic functions in two variables. Remaining after the expansion of the function in this case is lowered by two orders of magnitude. The use of such devices allows for easy synthesis, however, does not always produce the desired result, as in the structure already contains three of the distributor. When the expansion of the function using the module \[ a = f_0(1, 1), b = f_2(1, 0), c = f_1(0, 1), d = f_3(0, 0). \]

In order to bring the formula (1) to form (2) should be compared \( y_3 = f_3(1, 0) \), and \( y_4 = \overline{x} f_0(0, 0) + x f_1(1, 1) + \overline{x} x f_2(1, 0). \)

It is easy to verify that when substituted into the formula (1) for the corresponding values \( y_3 \) and \( y_4 \) of formula (1) and (2) coincide. Thus, the synthesis scheme in this case reduces to the determination of residual functions for inputs \( y_3 \) and \( y_4 \) and natural selection variable expansions for inputs \( x \) and \( y \).

The equation for the output of the module is as follows:

\[ z = (\overline{x} x + x) y_1 + x \overline{x} y_3 + \overline{x} x y_2. \] (3)

In order to bring the formula (3) to (2), should be compared \( y_3 = f_3(0, 0) \), \( y_2 = f_2(0, 1) \), \( y_1 = f_1(1, 1) + \overline{x} f_0(0, 0) \). It is easy to show that in this case the functions (2) and (3) coincide. Naturally there residual function is easier than for the decomposition in the case of formula (1). Furthermore, if the residual function \( f_3(0, 0) = f_1(1, 1) \), then \( y_1 = f_3(0, 0) = f_1(1, 1) \) implemented without additional logical operations, which greatly simplifies the residual function.

Thus, the conducted studies showed the strict compliance of the scientific direction "Synthesis of minimum control schemes of hydraulic and pneumatic drive systems" developed by the author with the point of view of general algebra, algebra of logic, graph theory and automata theory. The synthesis of the minimum graph of operations, which is a mathematical model of the control system, has been proved. The legitimacy of the methods of undivided decomposition of equations describing the scheme of the control system has been proved. The following literature may be interesting to read [8–12].

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